

Counter argument against Archimedes theory

Copyright © 2007 Mohammad-Reza Mehdinia

All rights reserved.

Contents

1. Explanations and Links.	Page 1-2
2. Facts:	Page 3-4
3. Counter argument against Archimedes theory!	Pages 5-17
4. Formulae.	Pages 18-24.

Explanations:

Standard circle = When a circles diameter is equal to the side of a square.

Other value = Values other than 3.125 for a circle.

Other circle = Circles with other values than 3.125.

See the book = “The Correct Values for π ”. This book is not available on the website.

See the internet version of the book “[The correct values for circle](#)” = this version is only about pi. Pages 44-48 of this book are illustrated on pages 18-24.

There are five links on the main page about pi which we sometimes refer to:

Links

1. [Click here](#): To view the book.
2. [Click here](#): To see the Square-squaring form.
3. [Click here](#): [Argument against Archimedes theory and others](#).
4. [Click here](#): Archimedes triangle theory and triangle angles and more.
Here I present “the angles” for polygons.
5. [Click here](#): To see the explanation of Sine and Cosine.

In my book I have defined values with the help of a standard circle. There are five values with which one can calculate circles. These values are very important to remember. With any chosen value one can calculate a circle. Once you have completely understood these values you will have a profound understanding about circles.

A standard circle is a circle positioned inside a square; this means that its diameter is equal to the side of the square. Every square has such a circle. The circumference of such circle **cuts** its squares perimeter in 4 points which is exactly in the middle of the squares sides. This circle is obtained with the value 3.125.

If we were to use other values the circle will not cut the squares sides or it will cut the squares perimeter in 8 points. The circles diameter will be longer or shorter. You can compare and see that it is the same for a standard sphere and its cylinder, a standard cub and its sphere and a standard cylinder and its cub.

See the book “The correct values for π ” or the version on the website “[The correct values for circle](#)” on pages 106-141 for definition, figures and formulae (ANALYSIS IN PRACTICE AND APPLICATION VALUES AND DIAGRAMS).

Important notice! Before you begin to study section 2 (Counter argument against

Archimedes theory) use the formula $(\ln \sqrt{s^2 * 2} / \ln s)^2 * 2 = 4Q$. The “s” in the formula is the side of a square used. Choose different squares for demonstration. Consider the results before you start to reading section 2.

Facts:

Until now the value 3.141... has been used by man for further development. There are and have been many theories and values about how to calculate a circle for example: 3.0, 3.20, 3.30, 3.14, 3.141, 3.1415, 3.1416, and 3.1417 etc. Any value could be used instead 3.141. Until now you know just one value and it is 3.141... I will introduce four more values which can be used. Referring to the square group one, two and three, these values belongs to 3.141... which is in group one. When using these values they will give you the same result.

- | | | |
|-----------------|---|--|
| 1. 3.141592653 | } | These will give an approximate result. |
| 2. 1.273239545 | | |
| 3. 0.7853981633 | | |
| 4. 0.6366197724 | | |
| 5. 0.2146018367 | | |

See pages 44-48 in the book and formulae number 19.

I will now describe the circle and its correct values. The new theory shows the values and their exercises. With the help of the values one can calculate the correct area and the correct circumference rather more easily than before.

The values in group two are:

- | | | |
|------------|---|--|
| 1. 3.125 | } | These will give the correct values for circle. |
| 2. 1.28 | | |
| 3. 0.78125 | | |
| 4. 0.64 | | |
| 5. 0.21875 | | |

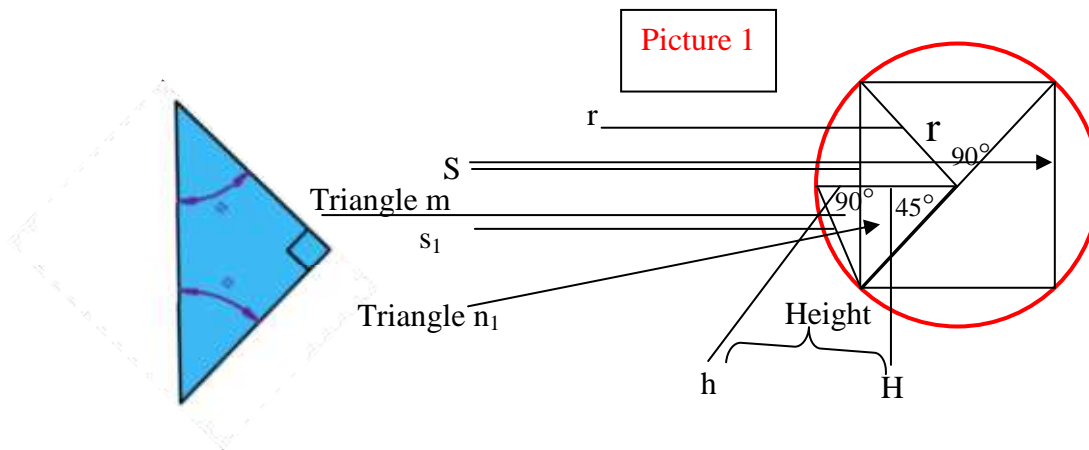
See pages 44-48 in the book and formulae 9.

Group two is called the **squaring form**. The square squaring-form is the only square that gives a circle that its diameter is equal to the side of square.

We have already got to know the values in group one,"[3.141592653](#), [1.273239545](#), [0.7853981633](#), [0.6366197724](#), [0.2146018367](#)". In this group, the circles diameter is bigger then its squares side. Further, if we use 3.141... instead of the value 3.125, all diameters will actually be bigger.

Now you understand that according to my theory when using 3.141... the circumference and the area of the circle is always bigger than the real circle.

Do not draw a circle round the square, this will only mislead you see picture 1.



You do not need to draw a circle around a square as Archimedes suggests. If you do this as the picture above, you get the wrong result. This has nothing to do with radius and circle.

You only need a square or a triangle to build a polygon as the figures 9-13 and 25, it tells you nothing about a circle!

Archimedes theory and all other theories do not define the circle.

The only way to calculate a correct value for a circle is introduced in the book

“The Correct Values for circle”.

Also with the formulae you can calculate chords. For further details see chapter chords in the

book “The Correct Values for π ”. In this chapter you can read how one can calculate

a chord via a straight line and vice versa.

Counter argument against Archimedes theory!

Archimedes polygon theory does not obtain all the steps necessary for the calculation of polygons. See [link four](#), chapter two. In this chapter you can only follow steps 1, 2 and 3 for polygons. In [Link 4](#): “Archimedes triangle theory” I present the “angles” for polygons.

Here we use Archimedes theory.

The polygon in step1 has an angle of 90° , in step 2 an angle of 135° and in step 3 an angle of 157.5° . When we continue to step 4 the polygon's circumference has a gap between the first polygon's side and the last polygon's side. **Due to this gap we can not call it a polygon, but just to clarify we call it an imaginary polygon.** The angle in step 4 is 168.75° , when we try to make a polygon with this angle we get the same result as the one mentioned above, there is a gap between the first and the last side of the polygon.

One can see that as the number of the triangles increases the gap becomes bigger, [figures 14 and 15, 16, 17](#).

As we continue down the steps the gap will get bigger and bigger. The bases of the triangles will form a staircase. The reason for this is that the bases of the triangles are straight lines and not chords; because of this the gap arises. The base of the triangles that now has formed a staircase will move towards to become a straight line, [figure 18-22](#). As we follow the steps the gap will get bigger. We can note that when the triangles reach angle 180° , the triangles stops to exist. The result will now be a straight line which is half of the square's diagonal, [figure 19-22 and 23](#). For illustrations see [figure 24](#) figure below.

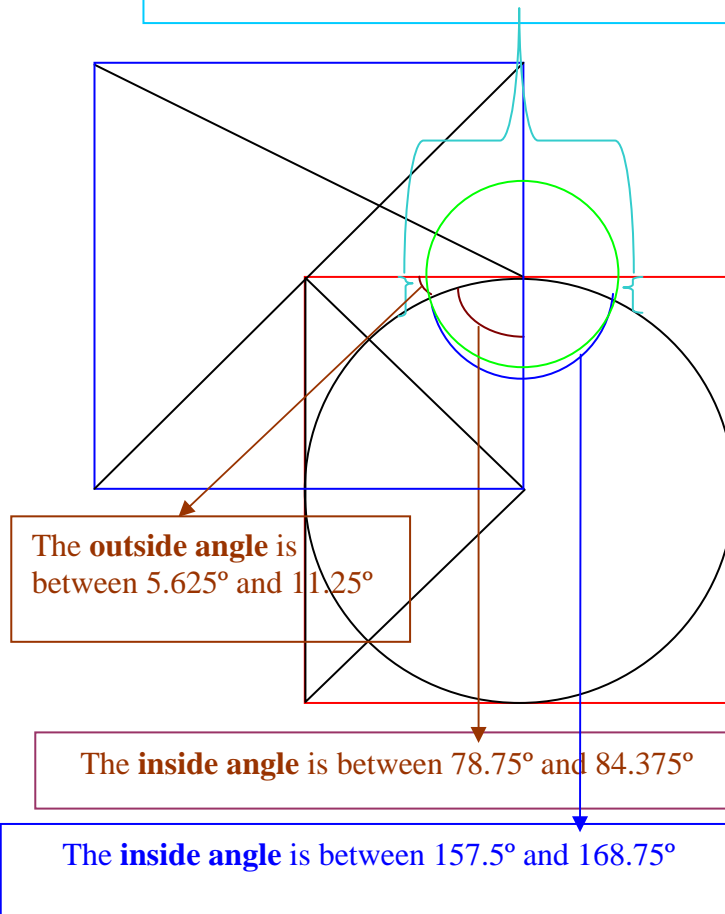
Here I will describe specific angles which are of great importance for ones correct understanding of the formulae.

See the figure below and observe what I mean with inside angle and outside angle for circles!

Figure 1

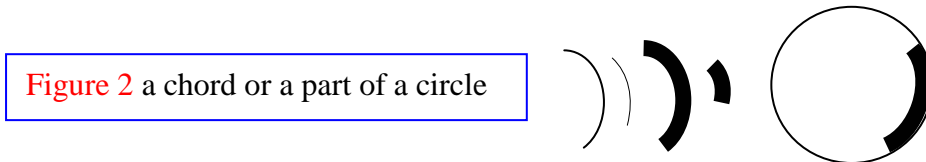
Observe outside angle of the circle and inside angle of the circle!

The **outside angles** are between 11.25° and 22.5°

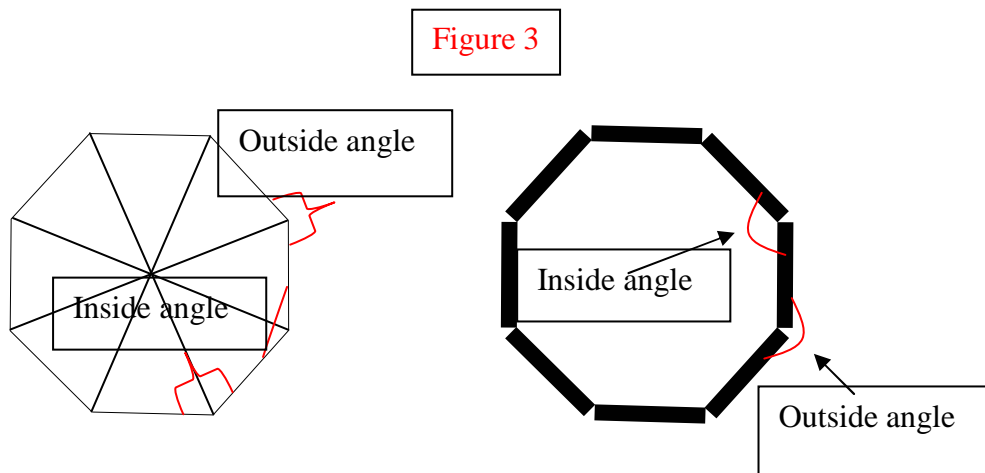


See **figure 1** and **figure 2**, where a chord's inside- and outside angle has not changed. The inside angle is between 78.75° and 84.375° and the outside angle is between 5.625° and 11.25° . Remember that a circle's angle never changes; see figure 1.

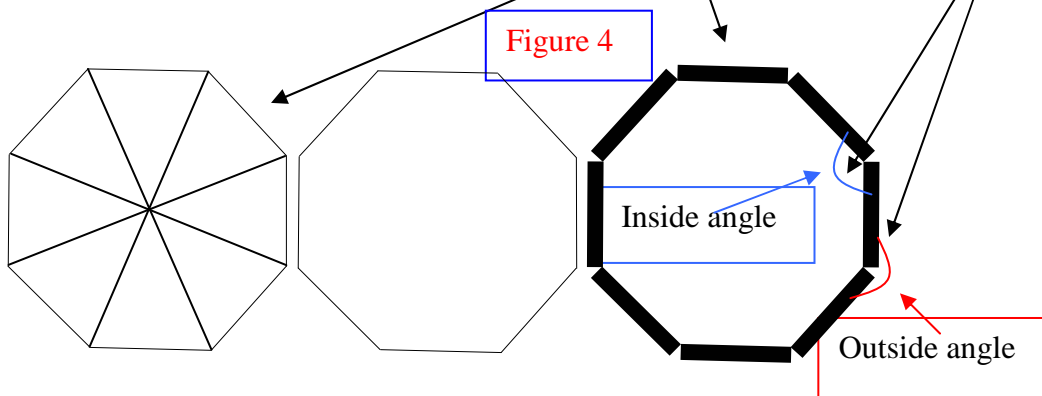
We have marked the thin chords **fat** for a better view.



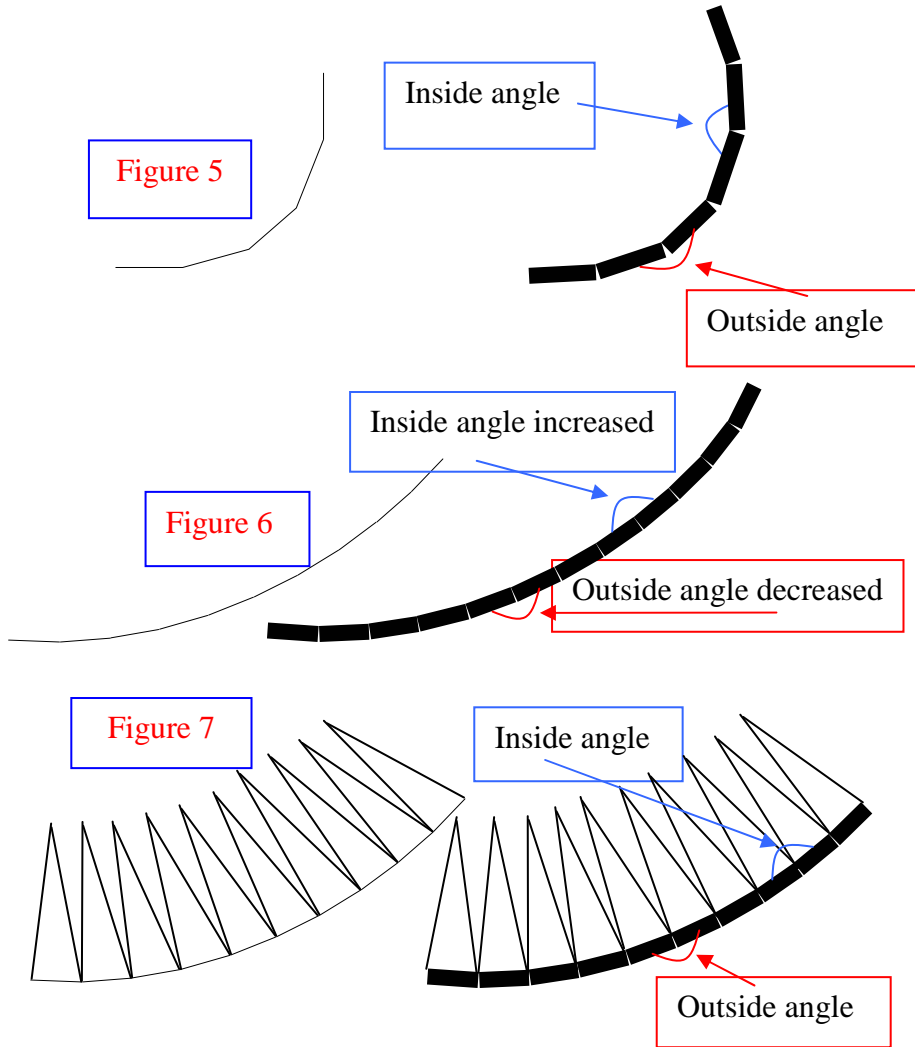
See **figure 3** below and observe what I mean with inside angle and outside angle for **polygons** and **imaginary polygons** in figure 5-13, se below!



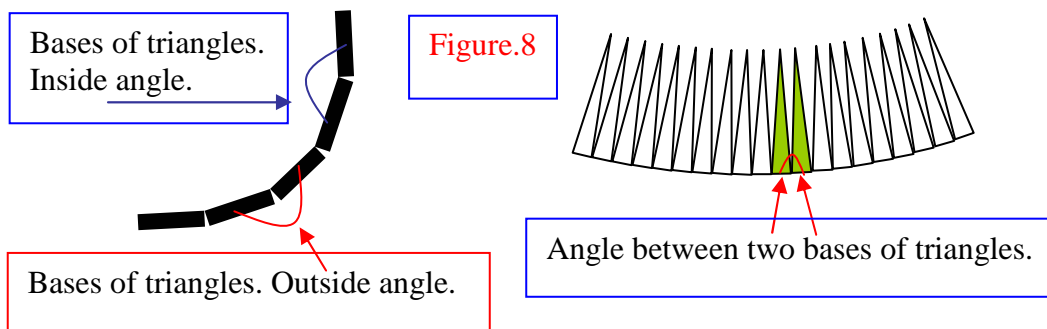
Look at the **figures 4, 5, 6, 7** below. These are triangle bases, a polygon has two angles, an inside angle and an outside angle, see figure 4.



When the inside angle increases the outside angle decreases; see figures 5, 6 and 7.



The gap forms when the inside angle increases the outside angle decreases. As the inside angle increases and the outside angle decreases the gap will become bigger. As this continues the triangles with their bases form a straight line, see figures 9-24 and 5-8 for an example.



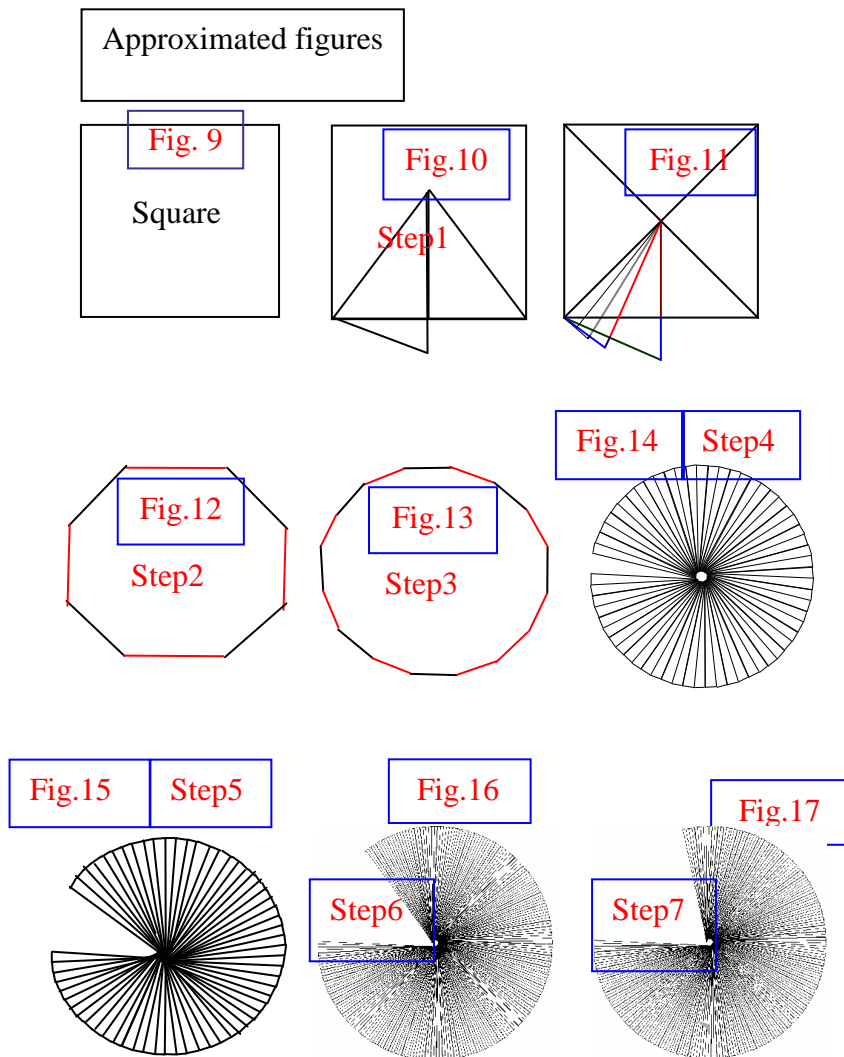
Archimedes and other theories are based on circles

but

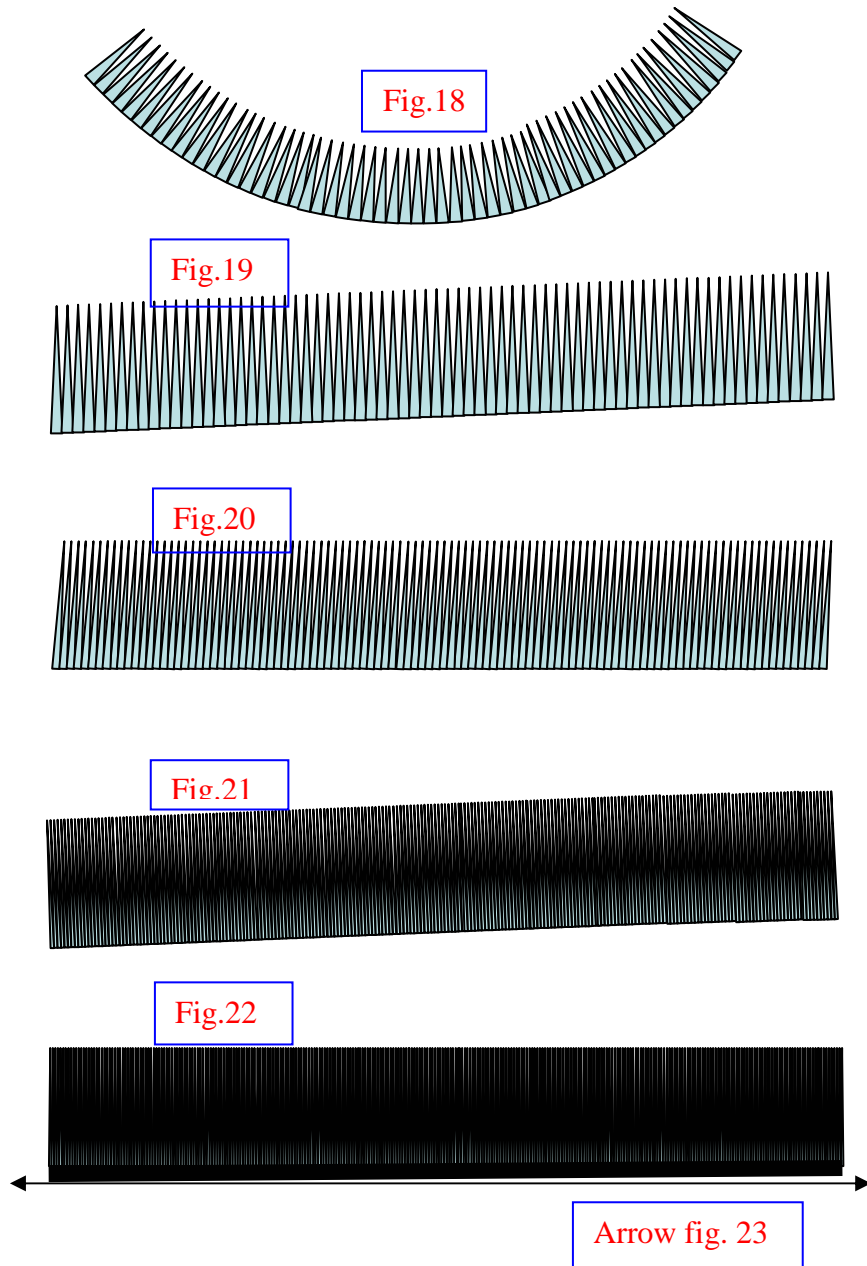
can not be explained with circles.

When using Archimedes theory you imagine that you get polygons BUT this is NOT correct. As explained above once you reach the fourth polygon a small gap appears. As the number of triangles increases the gap becomes bigger, this will result in that the triangle bases position themselves next to each other as in figure 18-19. As you continue to increase the number of the triangles as in figure 20, 21 and 22 the base of the triangles will eventually form a straight line. Also when the height of triangle reaches its hypotenuse the only thing that remains is a straight line.

Look at the figures 9–24 for an overview.



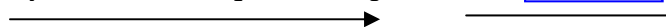
See the below figures how the gap becomes bigger and triangles bases decreases. The triangles are inclined to stay beside each others; finally together the bases of the triangles will approach to form a straight line. See figure 18-22.



3.141... with thousands triangles or decimals give figure 22.

The arrow figure 23 under the figure 22 is a straight line.

Finally a half of the square's diagonal



This is how Archimedes defined his theory.

Finally when you reach 180° the triangles disappear. Now you only have one half of the square's diagonal which is like a hypotenuse and not a radius, see figure 10 and 24!

The theory started with a hypotenuse and ended with a hypotenuse.

The straight line is what you have when the angle is 180°, which is half of the squares diagonal!

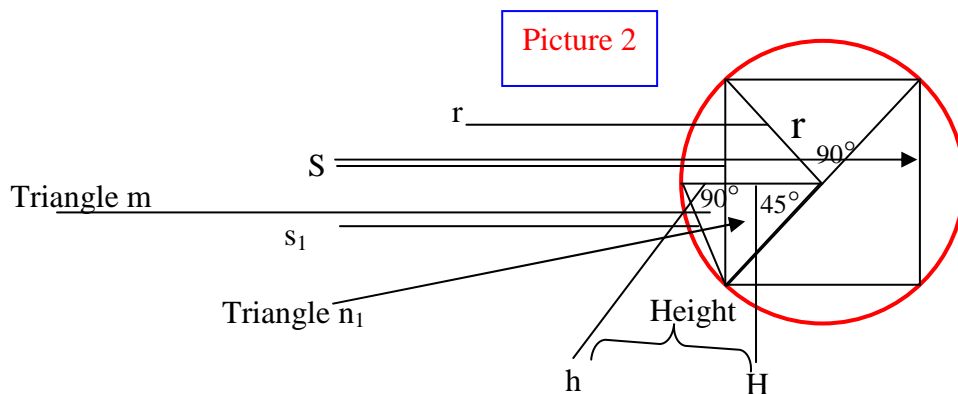
If one can introduce a formula or formulae which gives oss the angle between a cord and a straight line and define a circle as in figure 1, the formulae will be correct. If not so, the formulae like Archimedes theory is uncorrect.

[What is the angle when pi has million or billion decimals?](#)

Note in Archimedes theory that the sides had not halved, they are different; see the numbers underlined page 15-16!

Note again: picture 2 shows the method for constructing polygons via a square and triangles as Archimedes did. This method is not suitable for circles.

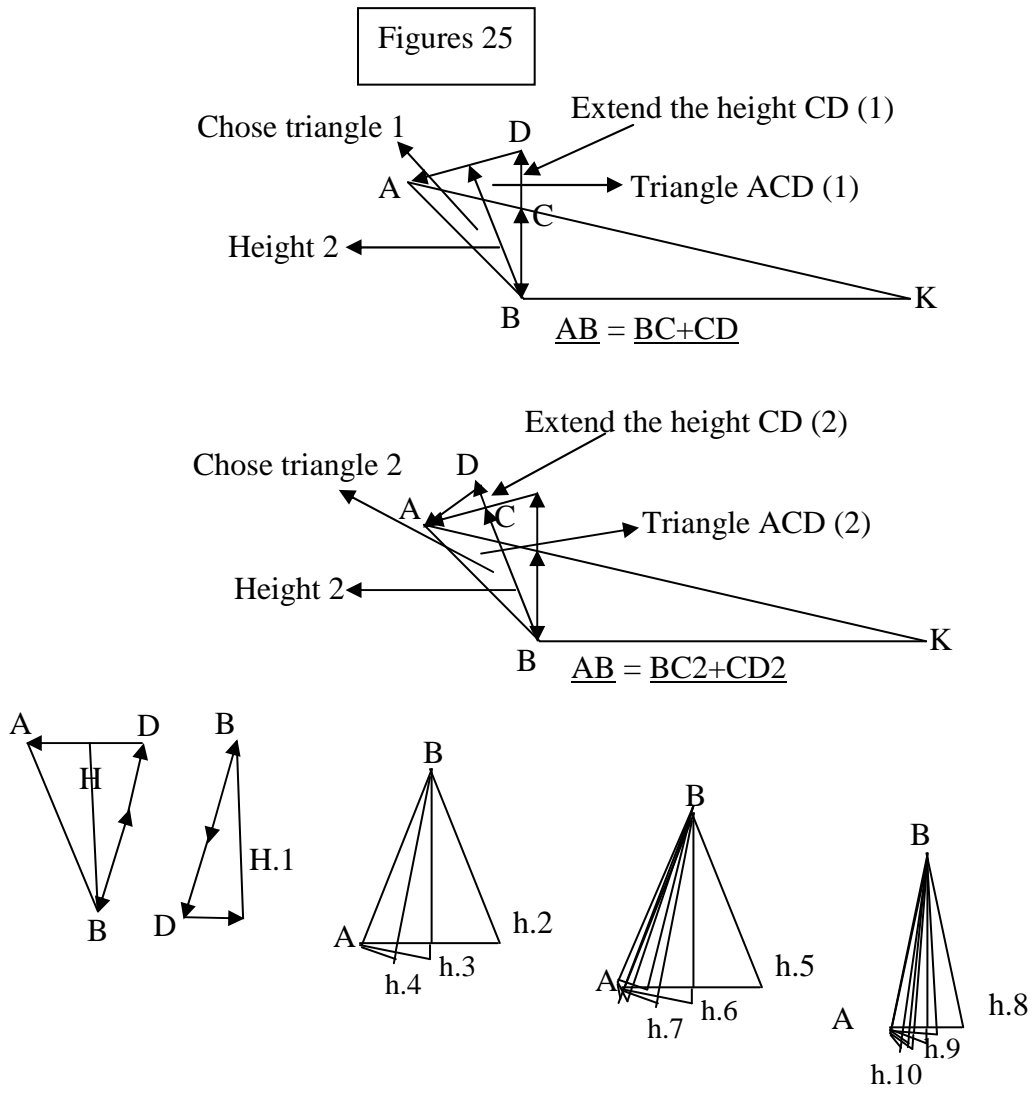
Referring to the [Triangle link 4](#):



We observe in step 13: there are 16384 triangles and the angle between two bases is 179.9780273°.

178.59375°	=====
179.9780273°	===== <u>Angle is on step 13.</u> =====
180°	=====

Look at the figures below, they can help you to get a perspective of the theory. Note that, when you use a triangle it is a part of a square and nothing else.



Different results with 3.141...

Using 3.141... according to my theory and according to Archimedes theory will give you different results and also has different definitions.

We know that Archimedes presented the value 3.141... with its polygon definition.

The counter argument above has now showed that Archimedes theory is not correct.

When using 3.141... it will give you a circle that cuts the squares perimeter in 8 points. A circle, a sphere, a cylinder and a cub calculated with 3.141... can not be placed inside each other; this means that you can not calculate the volume as explained in the book “The correct values for circle”.

For further information about 3.141... I refer to the following pages 44-48, 83, 90, 128-129, 155-157, 160-163 in the internet version of the book “The correct values for circle” or the book “The Correct Values for π ”.

In practice:

Draw a square. Use the side of the square as the circles diameter and calculate its circumference with value 3.141. Now position the circle into the square. One can see that the circle cuts the square’s perimeter in 8 points. This shows that the circles diameter is longer then the squares side when we use 3.141. This also means that the circle’s circumference is longer and area bigger, which it would not be the case if we were to use a standard circle. However, referring to my theory, any chosen value used in the formulae will help you to calculate the correct circle, diameter, circumference and area.

With the formulae you can also calculate a chord to straight line or vice versa, for further details see the book “The correct values for π ”.

The definition for Archimedes method is as described above.

Referring to the evidence presented in the book “The Correct Values for π ”, one can see that the value 3.125 is far more correct than 3.141... when calculating circles. There is no other way to define a circle.

According to my theory we get the wrong result when using 3.141... but even 3.141... has its circle when using the values I have stated in the book.

With polygons and triangles you can never construct a circle. If you do, the circle becomes bigger or smaller than planned. Therefore, you can never calculate a circle with the help of polygons or triangles. See Link 4, chapters 1 and 2-6 and Link 5, Sine & Cosine.

Some polygons are smaller than the circle and some polygons are bigger than the circle, because of the gap, in this way one can never build a circle.

The only way to calculate a correct value for a circle is introduced in the book

“The Correct Values for π ”.

To construct a polygon you must have a constant which you have received through triangles.

The information this constant gives us is about the polygons sides, length and its “**angle**”.

Note that the side of the square used in the beginning determines the length of the polygons side and the bases of the triangles.

With an equal sided polygon means that all the sides have the same length.

Note that the **angles** used below are the inside angles of the polygons. The outside angles are for you to solve.

1. Select a square with the side $\sqrt{2}$ u.l. Now we follow Archimedes first step which gives us a polygon with 8 sides. Lengths of all the sides are equal, 0.7653668647 u.l. The **angle** 135° and the value 3.061467459. With this value it is possible to calculate the area of the polygon and the numbers of sides. Here the polygon is smaller than the circle.
2. As we reach step seven we have a 256 sided imaginary polygon with each **angle** 178.59375° and side 0.0245430766 u.l.
3. In step thirteen the imaginary polygon has 16384 sides. All the sides have the same length. The side is 0.0003834951946 u.l, the **angle** is 179.9780273° and the value 3.141592653. This value gives the length of sides, the number of sides and the angle between the two sides. This imaginary polygon is now bigger than the circle.

Step 7: a 256 sided polygon, each angle 178.59375° is bigger than the circle. The imaginary polygon's side = 0.0245430766 , and the gap is bigger than step six.

As the step continues the gap becomes bigger until it reaches its full-size, see [figures 9-22](#).

As one can see the polygons stays bigger then the circle when the number of imaginary polygon's side increases. "Note that in step 4 there is an alteration" see area and circumference.

Even look at the Link 4, chapter 2 and figures 14:

See the link for triangles in chapter 2; now try to construct a polygon as described in this chapter. For more information see Link 4, chapters 3, 4, 5 and 6.

How to construct a polygon in the way that was described earlier:

1. When you are at step 3 the polygon is smaller than the standard circle. Note, the angle between the two sides is 157.5° , use this angle and draw a polygon with equal sides.
2. When you are at step 4 the polygons **area** is smaller than the standard circle. Note, the angle between the two sides is 168.75° , use this angle and draw a polygon with equal sides.
When you are at step 4 the circumference of the polygon is bigger than the standard circle. Note the angle between the two sides is 168.75° . Now make a polygon with equal sides. Now you will discover the gap I mentioned earlier.
3. When you are at step 5 the polygon is bigger than the circle. Note the angle between the two sides is 174.375° . Now make a polygon with equal sides.
4. When you are at step 13, the polygon is again bigger than the circle. The angle between two sides is 179.99780273° . Now draw this polygon and discover the gap.
5. And so forth.

Try to draw a polygon in the way that is introduced in "Archimedes Triangle theory". You can even select another side length. When you reach step 4, 5, 6... you will discover the gap which becomes bigger as the steps continues. Once you reach step 13 the imaginary polygon

will have 16384 sides and the angle between the two sides is $179.9780273^\circ \rightarrow \approx 180^\circ$. Draw this polygon. It is impossible to draw a polygon with this number of sides and angle.

If you succeed to have the height of the triangle the same as its hypotenuse the angles will be zero (0°). This means that the triangle and the polygon stops to exist.

The difference between 3.125 and 3.141 is 0.016..., who can possibly measure its difference in practice? For comments see chapter 3; pages 80-91 for cylinders in the book

“The correct values for a circle”.

The value you get through triangles, tells you how many sides the polygon has and its length. Remember, this does not tell you that this is a correct value for the standard circle. For further information please look at the [link 4](#), Archimedes Triangle Theory

Practice:

In practice you can see that 3.141 is bigger than the correct number. You must be very precise when calculating this value.

Practice:

1. Chose a radius and draw a circle.
2. Calculate its circumference with 3.141.
3. Make a plate with the circle with same radius. Then draw this circle again.
4. Prepare different threads.
5. Cut out the threads the same as the circumference you have calculated with 3.141.
6. Put the plate on the circle you have drawn so that the plate is covering the circle you have drawn.
7. Encircle one of the threads round the plate.
8. See their differences.

You can never come to a correct circle through polygons. The result you get will only be an approximated circle.

To avoid misinterpretation do not draw a circle around the square, for further information see the section for triangles.

Here are two different definitions for 3.141592683...

1. Triangles on a straight line see pages 1-17.
2. Circle with the formulae below.

When one uses a value to calculate a circle according to my theory one will exactly know from where the value is obtained and also the circle's diameter, circumference and area. I refer to pages 66-77 and 134-138 for detailed explanations of the correct and standard circle 3.125.

Note the square with side 3.928105676 in the formula gives the number 3.141595326.

There is no doubt and no hesitation in the formulae I have presented.

If you want to calculate a circle there is only one course of action, see the formulae below:

We show the pages 44-48 of the book below!

Squares formulae

$$Q = (\ln \sqrt{(e^{\ln s})^2 * 2} / \ln e^{\ln s})^2 / 2.$$

$$(\ln \sqrt{s^2 * 2} / \ln s)^2 / 2 = Q$$

$$(\ln \sqrt{s^2 * 2} / \ln s)^2 * 2 = 4Q$$

$$(\ln s / \ln \sqrt{s^2 * 2})^2 = r$$

$$(\ln s / \ln \sqrt{s^2 * 2})^2 * 2 = 2r$$

$$1 - (\ln \sqrt{s^2 * 2} / \ln s)^2 / 2 = b$$

$$1 - (\ln s / \ln \sqrt{s^2 * 2})^2 = h$$

See pages 44-48 in the book! Compare values and figures on pages 98-127.

Practice and Applications pages 92-97 and chapter five; diagrams on pages 98-127.

Observe that on pages 44-48 are 27 different values on every page. All number 1 formulae are obtained from one square. With these you can calculate a circle, but not a standard circle.

This correspond for all the formulae, number 2 formulae has the same square, number 3 the same and so forth. Number 19 formulae belong to square 3.928105767 which give the results 3.141592653, 0.785398163, 1.273239545, 0.6366197724 and 0.2146018367. See below. You

can choose any square you prefer to put in the formulae. It will give you five different values for a circle.

Here I have only listed the formulae for 27 squares but this is of course infinite.

The formulae below are named:

1. Relevant formula.
2. Percentage formula.
3. Inclusive formula.
4. Executive formula.
5. Shadows formula.

The formulae are useful!

Note: There are five universal values for squares and circles. These values can be used to calculate a circle. For details and application of these values look at pages 92-97-127 “ANALYSIS IN PRACTICE AND APPLICATION VALUES AND DIAGRAMS”.

The standard value for calculation of a circle is 3.125, when using other values the diameter of the circle will either be too long or too short. For further details regarding 3.141... see pages 114-116 in the book and below.

No.	$M = (\ln\sqrt{(e^{\ln 5})^2 * 2} / \ln e^{\ln 5})^2 * 2$. Relationship between diameter, circumference & area
1	$(\ln\sqrt{10^2 * 2} / \ln 10)^2 * 2 = 2.6473695... = 4Q < 4M$
2	$(\ln\sqrt{9^2 * 2} / \ln 9)^2 * 2 = 2.68068879... = 4Q < 4M$
3	$(\ln\sqrt{8^2 * 2} / \ln 8)^2 * 2 = 2.7222... = 4Q < 4M$
4	$(\ln\sqrt{7^2 * 2} / \ln 7)^2 * 2 = 2.77585615... = 4Q < 4M$
5	$(\ln\sqrt{6^2 * 2} / \ln 6)^2 * 2 = 2.84853316... = 4Q < 4M$
6	$(\ln\sqrt{5^2 * 2} / \ln 5)^2 * 2 = 2.95409426... = 4Q < 4M$
7	$(\ln\sqrt{4.5^2 * 2} / \ln 4.5)^2 * 2 = 3.027880092... = 4Q < 4M$
8	$(\ln\sqrt{4.1^2 * 2} / \ln 4.1)^2 * 2 = 3.103162981... = 4Q < 4M$
9	$(\ln\sqrt{4^2 * 2} / \ln 4)^2 * 2 = 3.125 = 4M$
10	$(\ln\sqrt{3.999^2 * 2} / \ln 3.999)^2 * 2 = 3.125225493... = 4Q > 4M$
11	$(\ln\sqrt{3.99^2 * 2} / \ln 3.99)^2 * 2 = 3.127261525... = 4Q > 4M$
12	$(\ln\sqrt{3.98^2 * 2} / \ln 3.98)^2 * 2 = 3.129537779... = 4Q > 4M$
13	$(\ln\sqrt{3.97^2 * 2} / \ln 3.97)^2 * 2 = 3.131828911... = 4Q > 4M$
14	$(\ln\sqrt{3.96^2 * 2} / \ln 3.96)^2 * 2 = 3.134135076... = 4Q > 4M$
15	$(\ln\sqrt{3.95^2 * 2} / \ln 3.95)^2 * 2 = 3.136456429... = 4Q > 4M$
16	$(\ln\sqrt{3.94^2 * 2} / \ln 3.94)^2 * 2 = 3.138793128... = 4Q > 4M$
17	$(\ln\sqrt{3.93^2 * 2} / \ln 3.93)^2 * 2 = 3.141145331... = 4Q > 4M$
18	$(\ln\sqrt{3.929^2 * 2} / \ln 3.929)^2 * 2 = 3.141381410... = 4Q > 4M$
19	$(\ln\sqrt{3.928105767^2 * 2} / \ln 3.928105767)^2 * 2 = 3.141592653... = (\pi) = 4Q > 4M$
20	$(\ln\sqrt{3.92^2 * 2} / \ln 3.92)^2 * 2 = 3.143513202... = 4Q > 4M$
21	$(\ln\sqrt{3.9^2 * 2} / \ln 3.9)^2 * 2 = 3.1482966603... = 4Q > 4M$
22	$(\ln\sqrt{3.5^2 * 2} / \ln 3.5)^2 * 2 = 3.259657005... = 4Q > 4M$
23	$(\ln\sqrt{3^2 * 2} / \ln 3)^2 * 2 = 3.46089568... = 4Q > 4M$
24	$(\ln\sqrt{2.5^2 * 2} / \ln 2.5)^2 * 2 = 3.799065628... = 4Q > 4M$
25	$(\ln\sqrt{2^2 * 2} / \ln 2)^2 * 2 = 4.5 = 4Q > 4M$
26	$(\ln\sqrt{1.5^2 * 2} / \ln 1.5)^2 * 2 = 6.8802370... = 4Q > 4M$
27	$(\ln\sqrt{1.4^2 * 2} / \ln 1.4)^2 * 2 = 8.24197343... = 4Q > 4M$

Relevant formulae

Group three

Group two only

Group one

No.	$M = (\ln\sqrt{(e^{\ln s})^2 * 2} / \ln e^{\ln s})^2 / 2.$	Percentage relationship between square & circle
1	$(\ln\sqrt{10^2 * 2} / \ln 10)^2 / 2 = 0.6618423801...Q < M$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">Percentage formulae</div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">Group three</div>
2	$(\ln\sqrt{9^2 * 2} / \ln 9)^2 / 2 = 0.6701721995...Q < M$	
3	$(\ln\sqrt{8^2 * 2} / \ln 8)^2 / 2 = 0.6805555556...Q < M$	
4	$(\ln\sqrt{7^2 * 2} / \ln 7)^2 / 2 = 0.6939640386...Q < M$	
5	$(\ln\sqrt{6^2 * 2} / \ln 6)^2 / 2 = 0.712133332904...Q < M$	
6	$(\ln\sqrt{5^2 * 2} / \ln 5)^2 / 2 = 0.73852355662...Q < M$	
7	$(\ln\sqrt{4.5^2 * 2} / \ln 4.5)^2 / 2 = 0.756970023...Q < M$	
8	$(\ln\sqrt{4.1^2 * 2} / \ln 4.1)^2 / 2 = 0.7757907452...Q < M$	
9	$(\ln\sqrt{4^2 * 2} / \ln 4)^2 / 2 = 0.78125 = M$	Group two only
10	$(\ln\sqrt{3.999^2 * 2} / \ln 3.999)^2 / 2 = 0.7813062735...Q > M$	<div style="border: 1px solid black; padding: 5px; margin-top: 10px;">Group one</div>
11	$(\ln\sqrt{3.99^2 * 2} / \ln 3.99)^2 / 2 = 0.7818153813...Q > M$	
12	$(\ln\sqrt{3.98^2 * 2} / \ln 3.98)^2 / 2 = 0.7823844446...Q > M$	
13	$(\ln\sqrt{3.97^2 * 2} / \ln 3.97)^2 / 2 = 0.7829572278...Q > M$	
14	$(\ln\sqrt{3.96^2 * 2} / \ln 3.96)^2 / 2 = 0.7835337691...Q > M$	
15	$(\ln\sqrt{3.95^2 * 2} / \ln 3.95)^2 / 2 = 0.7841141073...Q > M$	
16	$(\ln\sqrt{3.94^2 * 2} / \ln 3.94)^2 / 2 = 0.7846982819...Q > M$	
17	$(\ln\sqrt{3.93^2 * 2} / \ln 3.93)^2 / 2 = 0.7852863328...Q > M$	
18	$(\ln\sqrt{3.929^2 * 2} / \ln 3.929)^2 / 2 = 0.7853453526...Q > M$	
19	$(\ln\sqrt{3.928105767^2 * 2} / \ln 3.928105767)^2 / 2 = 0.785398163... = (\pi/4) = Q > M$	
20	$(\ln\sqrt{3.92^2 * 2} / \ln 3.92)^2 / 2 = 0.7858783005...Q > M$	
21	$(\ln\sqrt{3.9^2 * 2} / \ln 3.9)^2 / 2 = 0.7870741511...Q > M$	
22	$(\ln\sqrt{3.5^2 * 2} / \ln 3.5)^2 / 2 = 0.8149142637...Q > M$	
23	$(\ln\sqrt{3^2 * 2} / \ln 3)^2 / 2 = 0.865223921...Q > M$	
24	$(\ln\sqrt{2.5^2 * 2} / \ln 2.5)^2 / 2 = 0.9497664071...Q > M$	
25	$(\ln\sqrt{2^2 * 2} / \ln 2)^2 / 2 = 1.125Q > M$	
26	$(\ln\sqrt{1.5^2 * 2} / \ln 1.5)^2 / 2 = 1.720059253...Q > M$	
27	$(\ln\sqrt{1.4^2 * 2} / \ln 1.4)^2 / 2 = 2.060493358...Q > M$	

$$M = (\ln e^{\ln 5} / \ln \sqrt{(e^{\ln 5})^2 * 2})^2 * 2.$$

No.	Relationship between diameter, circumference and area	
1	$(\ln 10 / \ln \sqrt{10^2 * 2})^2 * 2 = 1.510933766... = 2r > 2R$	Inclusive formulae
2	$(\ln 9 / \ln \sqrt{9^2 * 2})^2 * 2 = 1.492153809... = 2r > 2R$	
3	$(\ln 8 / \ln \sqrt{8^2 * 2})^2 * 2 = 1.469387755... = 2r > 2R$	
4	$(\ln 7 / \ln \sqrt{7^2 * 2})^2 * 2 = 1.440996859... = 2r > 2R$	
5	$(\ln 6 / \ln \sqrt{6^2 * 2})^2 * 2 = 1.404231502... = 2r > 2R$	
6	$(\ln 5 / \ln \sqrt{5^2 * 2})^2 * 2 = 1.354052932... = 2r > 2R$	
7	$(\ln 4.5 / \ln \sqrt{4.5^2 * 2})^2 * 2 = 1.321056276... = 2r > 2R$	
8	$(\ln 4.1 / \ln \sqrt{4.1^2 * 2})^2 * 2 = 1.289007385... = 2r > 2R$	
9	$(\ln 4 / \ln \sqrt{4^2 * 2})^2 * 2 = 1.28 = 2R$	Group two only
10	$(\ln 3.999 / \ln \sqrt{3.999^2 * 2})^2 * 2 = 1.2799076... = 2r < 2R$	Group one
11	$(\ln 3.99 / \ln \sqrt{3.99^2 * 2})^2 * 2 = 1.279074349... = 2r < 2R$	
12	$(\ln 3.98 / \ln \sqrt{3.98^2 * 2})^2 * 2 = 1.278144021... = 2r < 2R$	
13	$(\ln 3.97 / \ln \sqrt{3.97^2 * 2})^2 * 2 = 1.277208977... = 2r < 2R$	
14	$(\ln 3.96 / \ln \sqrt{3.96^2 * 2})^2 * 2 = 1.276269179... = 2r < 2R$	
15	$(\ln 3.95 / \ln \sqrt{3.95^2 * 2})^2 * 2 = 1.275324587... = 2r < 2R$	
16	$(\ln 3.94 / \ln \sqrt{3.94^2 * 2})^2 * 2 = 1.274375162... = 2r < 2R$	
17	$(\ln 3.93 / \ln \sqrt{3.93^2 * 2})^2 * 2 = 1.273420863... = 2r < 2R$	
18	$(\ln 3.929 / \ln \sqrt{3.929^2 * 2})^2 * 2 = 1.273325164... = 2r < 2R$	
19	$(\ln 3.928105767 / \ln \sqrt{3.928105767^2 * 2})^2 * 2 = 1.273239545... (2r \text{ of } \pi) < 2R$	
20	$(\ln 3.92 / \ln \sqrt{3.92^2 * 2})^2 * 2 = 1.272461651... = 2r < 2R$	
21	$(\ln 3.9 / \ln \sqrt{3.9^2 * 2})^2 * 2 = 1.270528321... = 2r < 2R$	
22	$(\ln 3.5 / \ln \sqrt{3.5^2 * 2})^2 * 2 = 1.227122956... = 2r < 2R$	
23	$(\ln 3 / \ln \sqrt{3^2 * 2})^2 * 2 = 1.155770172... = 2r < 2R$	
24	$(\ln 2.5 / \ln \sqrt{2.5^2 * 2})^2 * 2 = 1.052890471... = 2r < 2R$	
25	$(\ln 2 / \ln \sqrt{2^2 * 2})^2 * 2 = 0.888888888888... = 2r < 2R$	
26	$(\ln 1.5 / \ln \sqrt{1.5^2 * 2})^2 * 2 = 0.5813753209... = 2r < 2R$	
27	$(\ln 1.4 / \ln \sqrt{1.4^2 * 2})^2 * 2 = 0.4853206617... = 2r < 2R$	

No.	$R = (\ln e^{\ln s} / \ln \sqrt{(e^{\ln s})^2 * 2})^2$	Relationship between radius and side
1	$(\ln 10 / \ln \sqrt{10^2 * 2})^2 = 0.7554668831\dots$	$r > R$
2	$(\ln 9 / \ln \sqrt{9^2 * 2})^2 = 0.7460769044\dots$	$r > R$
3	$(\ln 8 / \ln \sqrt{8^2 * 2})^2 = 0.7346938776\dots$	$r > R$
4	$(\ln 7 / \ln \sqrt{7^2 * 2})^2 = 0.7204984296\dots$	$r > R$
5	$(\ln 6 / \ln \sqrt{6^2 * 2})^2 = 0.702115751\dots$	$r > R$
6	$(\ln 5 / \ln \sqrt{5^2 * 2})^2 = 0.6770264658\dots$	$r > R$
7	$(\ln 4.5 / \ln \sqrt{4.5^2 * 2})^2 = 0.6605281382\dots$	$r > R$
8	$(\ln 4.1 / \ln \sqrt{4.1^2 * 2})^2 = 0.6445036927\dots$	$r > R$
9	$(\ln 4 / \ln \sqrt{4^2 * 2})^2 = 0.64 = R$	$r = R$
10	$(\ln 3.999 / \ln \sqrt{3.999^2 * 2})^2 = 0.6399538222\dots$	$r < R$
11	$(\ln 3.99 / \ln \sqrt{3.99^2 * 2})^2 = 0.6395371746\dots$	$r < R$
12	$(\ln 3.98 / \ln \sqrt{3.98^2 * 2})^2 = 0.6390720105\dots$	$r < R$
13	$(\ln 3.97 / \ln \sqrt{3.97^2 * 2})^2 = 0.6386044885\dots$	$r < R$
14	$(\ln 3.96 / \ln \sqrt{3.96^2 * 2})^2 = 0.6381345894\dots$	$r < R$
15	$(\ln 3.95 / \ln \sqrt{3.95^2 * 2})^2 = 0.6376622935\dots$	$r < R$
16	$(\ln 3.94 / \ln \sqrt{3.94^2 * 2})^2 = 0.6371875809\dots$	$r < R$
17	$(\ln 3.93 / \ln \sqrt{3.93^2 * 2})^2 = 0.6367104317\dots$	$r < R$
18	$(\ln 3.929 / \ln \sqrt{3.929^2 * 2})^2 = 0.636662582\dots$	$r < R$
19	$(\ln 3.928105767 / \ln \sqrt{3.928105767^2 * 2})^2 = 0.6366197724\dots$	$(r \text{ of } \pi) < R$
20	$(\ln 3.92 / \ln \sqrt{3.92^2 * 2})^2 = 0.6362308257\dots$	$r < R$
21	$(\ln 3.9 / \ln \sqrt{3.9^2 * 2})^2 = 0.6352641607\dots$	$r < R$
22	$(\ln 3.5 / \ln \sqrt{3.5^2 * 2})^2 = 0.6135614779\dots$	$r < R$
23	$(\ln 3 / \ln \sqrt{3^2 * 2})^2 = 0.577885086\dots$	$r < R$
24	$(\ln 2.5 / \ln \sqrt{2.5^2 * 2})^2 = 0.5264452357\dots$	$r < R$
25	$(\ln 2 / \ln \sqrt{2^2 * 2})^2 = 0.4444444444\dots$	$r < R$
26	$(\ln 1.5 / \ln \sqrt{1.5^2 * 2})^2 = 0.2906876605\dots$	$r < R$
27	$(\ln 1.4 / \ln \sqrt{1.4^2 * 2})^2 = 0.2526603309\dots$	$r < R$

Executive formulae

Group three

Group two only

Group one

No.	$M = (\ln \sqrt{(e^{\ln S})^2 * 2 / \ln e^{\ln S}})^2 / 2$	Value for shaded area and shaded circumference
1	$1 - (\ln \sqrt{10^2 * 2 / \ln 10})^2 / 2 = 0.3381576199...$	b of 10 > B of 4
2	$1 - (\ln \sqrt{9^2 * 2 / \ln 9})^2 / 2 = 0.3298278005...$	b of 9 > B
3	$1 - (\ln \sqrt{8^2 * 2 / \ln 8})^2 / 2 = 0.3194444444...$	b of 8 > B
4	$1 - (\ln \sqrt{7^2 * 2 / \ln 7})^2 / 2 = 0.3060359614...$	b of 7 > B
5	$1 - (\ln \sqrt{6^2 * 2 / \ln 6})^2 / 2 = 0.2878667096...$	b of 6 > B
6	$1 - (\ln \sqrt{5^2 * 2 / \ln 5})^2 / 2 = 0.2614764338...$	b of 5 > B
7	$1 - (\ln \sqrt{4.5^2 * 2 / \ln 4.5})^2 / 2 = 0.243029977...$	b of 4.5 > B
8	$1 - (\ln \sqrt{4.1^2 * 2 / \ln 4.1})^2 / 2 = 0.2242092548...$	b of 4.1 > B
9	$1 - (\ln \sqrt{4^2 * 2 / \ln 4})^2 / 2 = 0.21875 = B$	B of 4
10	$1 - (\ln \sqrt{3.999^2 * 2 / \ln 3.999})^2 / 2 = 0.2186936265...$	b of 3.999 < B of 4
11	$1 - (\ln \sqrt{3.99^2 * 2 / \ln 3.99})^2 / 2 = 0.2181846187...$	b of 3.99 < B
12	$1 - (\ln \sqrt{3.98^2 * 2 / \ln 3.98})^2 / 2 = 0.2176155554...$	b of 3.98 < B
13	$1 - (\ln \sqrt{3.97^2 * 2 / \ln 3.97})^2 / 2 = 0.2170427722...$	b of 3.97 < B
14	$1 - (\ln \sqrt{3.96^2 * 2 / \ln 3.96})^2 / 2 = 0.2164662309...$	b of 3.96 < B
15	$1 - (\ln \sqrt{3.95^2 * 2 / \ln 3.95})^2 / 2 = 0.2158858927...$	b of 3.95 < B
16	$1 - (\ln \sqrt{3.94^2 * 2 / \ln 3.94})^2 / 2 = 0.2153017181...$	b of 3.94 < B
17	$1 - (\ln \sqrt{3.93^2 * 2 / \ln 3.93})^2 / 2 = 0.2147136672...$	b of 3.93 < B
18	$1 - (\ln \sqrt{3.929^2 * 2 / \ln 3.929})^2 / 2 = 0.2146546474...$	b of 3.929 < B
19	$1 - (\ln \sqrt{3.928...^2 * 2 / \ln 3.928...})^2 / 2 = 0.2146018367$	b of 3.928105767 of π < B
20	$1 - (\ln \sqrt{3.92^2 * 2 / \ln 3.92})^2 / 2 = 0.2141216995...$	b of 3.92 < B
21	$1 - (\ln \sqrt{3.9^2 * 2 / \ln 3.9})^2 / 2 = 0.2129258489...$	b of 3.9 < B
22	$1 - (\ln \sqrt{3.5^2 * 2 / \ln 3.5})^2 / 2 = 0.1850857363...$	b of 3.5 < B
23	$1 - (\ln \sqrt{3^2 * 2 / \ln 3})^2 / 2 = 0.134776079...$	b of 3 < B
24	$1 - (\ln \sqrt{2.5^2 * 2 / \ln 2.5})^2 / 2 = 0.1850857363...$	b of 2.5 < B
25	$1 - (\ln \sqrt{2^2 * 2 / \ln 2})^2 / 2 = -0.125$	b of 2 < B
26	$1 - (\ln \sqrt{1.5^2 * 2 / \ln 1.5})^2 / 2 = -0.7200592526...$	b of 1.5 < B
27	$1 - (\ln \sqrt{1.4^2 * 2 / \ln 1.4})^2 / 2 = -1.060493358$	b of 1.4 < B

Shadow formulae

Group three

Group two only

Group one

For further details see the chapter chords, in this chapter you can read how one can calculate a chord via a straight line and vice versa. “The Correct Values for π ”.